Deformation Modelling in Interreg 4A project IRFO - Intelligent Robots for Handling of Flexible Objects

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INTERREG4A
SYDDANMARK-SCHLESWIG-K.E.R.N.

European Union
European Regional
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Outline

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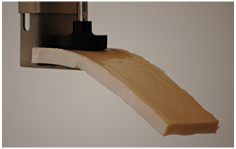
1. The need for Modelling
2. Modelling in context of IRFO system
3. Behaviour of Flexible Objects
4. Accomplished
5. Future plans

The need for Modelling

Rigid: Idealization, neglecting deformation

- The assumption of traditional robotics
- Keeps mathematical models simple.
- No such thing as a rigid object!
- An physical approximation for stiff

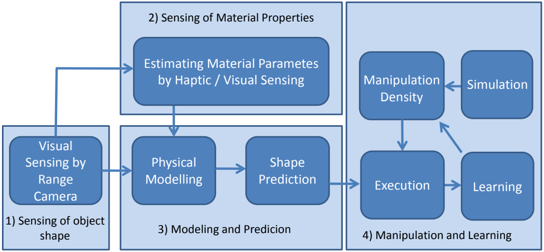
Importance of Modelling



When is something elastic?

- Always! - but rigid may be a very good approximation
- Relevant parameters
  - Scale
  - Forces
  - Length
  - Stiffness
  - Weight

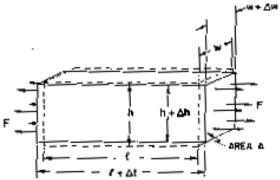
Modelling in the System



Modelling and Prediction

- Receives object shape data from stage 1) and material properties from stage 2)
- Predicts the shape and sends data to the Manipulation/Learning stage

Hooke's Law



Relating stress and strain

- $\frac{F}{A} = E \times \frac{\Delta l}{l}$
- Stress - force per unit area $\frac{F}{A}$
- Young's modulus E
- Strain - stretch per unit length $\frac{\Delta l}{l}$

Elasticity - A light introduction

What's going to happen

- We're making Hooke's Law $Stress = (Youngs\ Modulus) \times (Strain)$ work on a 3D elastic body for all sorts of crazy deformations, then we combine it with Newton's 2nd Law and end up with equations of motion
- In place of the simple stress (force) in Hooke's Law we're going to introduce the stress tensor, and in place of the simple strain (deformation) we're introducing the strain tensor

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The Stiffness/elasticity tensor

Generalizing Hooke's Law

- *Stresses are proportional to the strains*
- We must linearly relate each component of the stress tensor σ_{kl} to each component of the strain tensor u_{ik}

$$\sigma_{ij} = \sum_{k,l} C_{ijkl} u_{kl}$$

- where C_{ijkl} is the stiffness tensor
- Multi-dimensional analog to spring constant ($F = -kx$)

The Equations of Motion

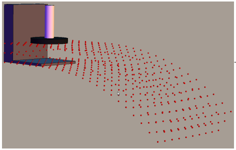
The equations of motion for an elastic material

- We skipped a few steps, where we revisited Newton's 2nd Law and our vector calculus
- For the important special case of a *homogeneous, isotropic* material we end up with

$$\rho \frac{\partial^2 \boldsymbol{u}}{\partial t^2} = (\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) + \mu \nabla^2 \boldsymbol{u} \tag{3}$$

- where λ and μ are the Lamé coefficients
- This PDE is the Navier-Cauchy equation

3D numerical solution to Navier-Cauchy PDE



$$
\begin{aligned}
f_x = (\lambda + \mu) \Bigg[& \frac{u_x(i+1,j,k) - 2u_x(i,j,k) + u_x(i-1,j,k)}{{h_x}^2} \\
& + \frac{u_y(i+1,j+1,k) - u_y(i+1,j-1,k) - u_y(i-1,j+1,k) + u_y(i-1,j-1,k)}{4h_x h_y} \\
& + \frac{u_z(i+1,j,k+1) - u_z(i+1,j,k-1) - u_z(i-1,j,k+1) + u_z(i-1,j,k-1)}{4h_x h_z} \Bigg] \\
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\end{aligned}
$$

Finite Difference Simulation software developed

- *An Outline for an intelligent System performing Peg-in-Hole Actions with flexible Objects, ICIRA 2011*
- + Supports relevant robotics grasping situations
- + Full 3D deformation output
- - Computationally expensive (seconds per frame), but large room for optimization, GPUs, etc.
- -Domain problems!

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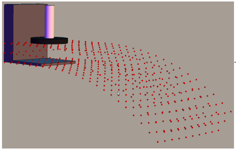

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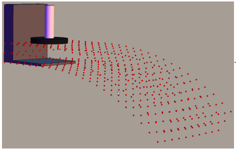

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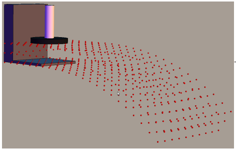
3D numerical instability

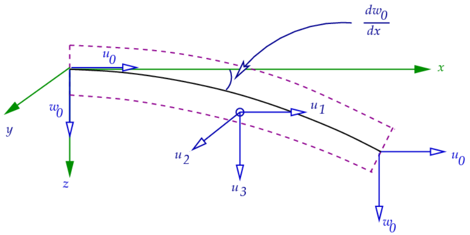


$$f_x = (\lambda + \mu)\left[\frac{u_x(i+1,j,k) - 2u_x(i,j,k) + u_x(i-1,j,k)}{{h_x}^2}\right.$$
$$+\frac{u_y(i+1,j+1,k) - u_y(i+1,j-1,k) - u_y(i-1,j+1,k) + u_y(i-1,j-1,k)}{4h_x h_y}$$
$$\left.+\frac{u_z(i+1,j,k+1) - u_z(i+1,j,k-1) - u_z(i-1,j,k+1) + u_z(i-1,j,k-1)}{4h_x h_z}\right]$$
$$+\mu\left[\frac{u_x(i+1,j,k) - 2u_x(i,j,k) + u_x(i-1,j,k)}{{h_x}^2}\right.$$
$$+\frac{u_x(i,j+1,k) - 2u_x(i,j,k) + u_x(i,j-1,k)}{{h_y}^2}$$
$$\left.+\frac{u_x(i,j,k+1) - 2u_x(i,j,k) + u_x(i,j,k-1)}{{h_z}^2}\right]$$

Domain

- Experiences: Soft materials + thin object = large strains/rotations: Numerical instability
- Stable up to deformation angles of around 30^o - Not sufficient for IRFO!
- Root cause: Linearizing simplification in theory (strain tensor)

Beam models



Beam models

- Bending deformation only
- Stretching in x-direction neglected
- Cross-section info (y and z) through second moment of inertia I.

Linear beam model

$$EI\,\frac{d^4w}{dx^4} = q(x) \tag{4}$$

Small deformation

- *Learning Peg-In-Hole Actions with flexible Objects, SSIR 2012*
- Same linear strain tensor as Navier-Cauchy PDE, but has analytical solutions which are inherently stable
- Geometric post-processing can compensate for these errors.

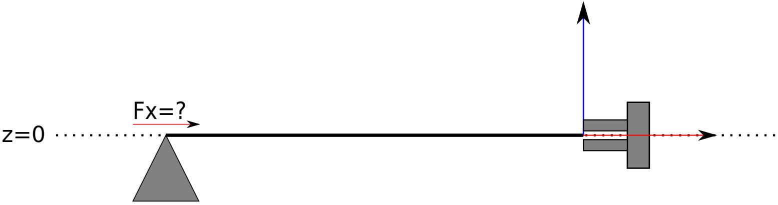
Non-linear beam model

$$EI\,\frac{d^4w}{dx^4} - \frac{3}{2}\,EA\left(\frac{dw}{dx}\right)^2\left(\frac{d^2w}{dx^2}\right) = q(x) \qquad (5)$$

Large-deformation

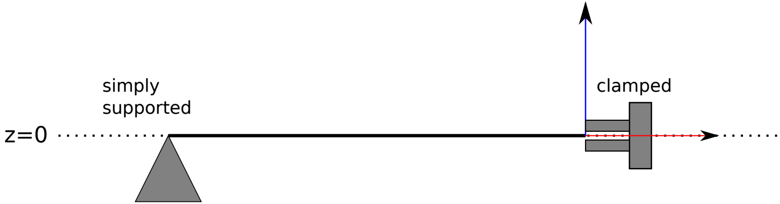
- Here shown for constant E, I and A
- Extra non-linear term (second order effects included)
- Numerical solution methods the only viable option

Laying Down



- The problem:
  - Laying down/Placement operation
  - Determine the unknown elastic force $F_x(x = 0, t)$ due to bending, at the table contact point.
  - Point is fixed due to static friction, but for some critical force F_{crit} it will begin to slide.

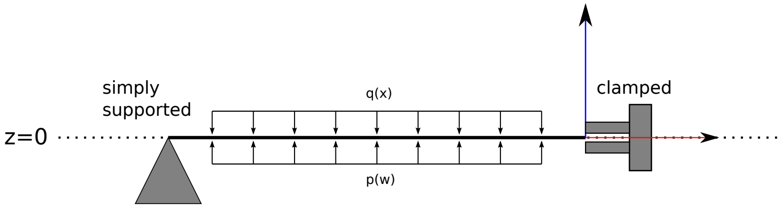
Laying Down



- Assumptions:
  - End at $x = 0$ is fixed on the table, but free to rotate
  - End at $x = L$ is grasped by the robot gripper, with a known position and rotation

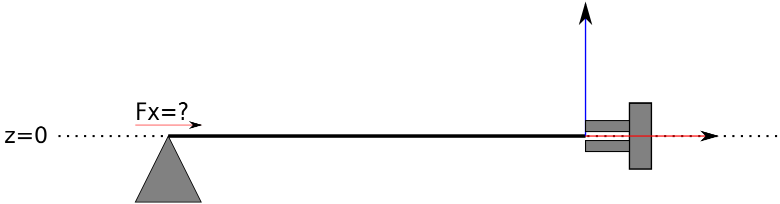
- Boundary conditions:
  - x=0: Simply supported, $w(0, t) = 0$, $w''(0, t) = 0$
  - x=L: Clamped: $w(L, t) = f(t)$, $w'(L, t) = g(t)$

Laying Down



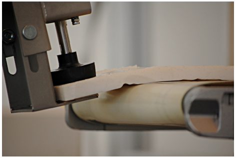
- Penalize table contact ($w < 0$):
  - $p(w) = F_p H(0 > w > -\varepsilon)$
  - $H(w)$ is the Heaviside step function
  - F_p is the penalty force
- Include as an extra source term:
  - E.g. $EI\frac{d^4w}{dx^4} + \ldots = q(x) + p(w)$
  - If not already, equation will become non-linear

Laying Down



- Having solved for the deformation $w(x, t)$ by the constitutive laws of elasticity, strain and geometric relations we obtain $F_x(x = 0, t)$.
- Should $F_x(x = 0, t)$ exceed some critical limit F_{crit}, the object will overcome static friction and begin to slide

Summary



- Grasping
  - *An Outline for an intelligent System performing Peg-in-Hole Actions with flexible Objects, ICIRA 2011*
- Peg-in-Hole
  - *Learning Peg-In-Hole Actions with flexible Objects, SSIR 2012*
- Placement/Laying down operations (Modelling and Learning)
  - Ongoing work, as presented
- Vision integration and estimation of parameters
  - Ongoing work

Thanks for your attention